

Trees Rules for Quantifiers and Identity (Phase 2)

PHIL 422

Woodbridge

Here are the quantifier rules for sentences where the main (or main after a negation) operator is a quantifier or the identity predicate.

Universal: $\forall x \Psi(x) \checkmark_n$ [provisionally marked – complete when all names on the branch are used in the result]
|
 $\Psi(n)$ [for any ‘ n ’]

Negated Universal: $\neg \forall x \Psi(x) \checkmark$
|
 $\neg \Psi(n)$ [‘ n ’ must be new to the branch]

Existential: $\exists x \Psi(x) \checkmark$
|
 $\Psi(n)$ [‘ n ’ must be new to the branch]

Negated Existential: $\neg \exists x \Psi(x) \checkmark_n$ [provisionally marked – complete when all names on the branch are used in the result]
|
 $\neg \Psi(n)$ [for any ‘ n ’]

Identity: $\Psi(n) \checkmark_m$ [checked off (even if a literal) to indicate substitution]
 $(n = m)$
|
 $\Psi(m)$ [m/n] [marked to indicate that ‘ m ’ has gone in for ‘ n ’]

Negated Identity: $\neg(n = n) / (n \neq n)$ [negation of an identity sentence using one name closes the branch]
 $\otimes$

Examples

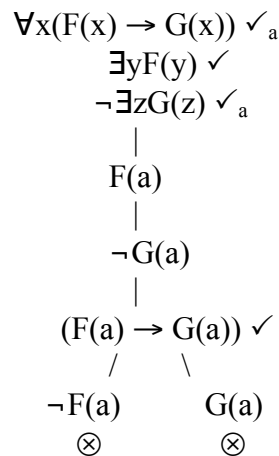
I. Show: $\{ \forall x(F(x) \rightarrow G(x)), \exists yF(y) \} \vdash \exists zG(z)$

Step 1: Construct Σ (which, for $\Gamma \vdash \phi$, is $\Gamma \cup \{\neg \phi\}$)
so $\Sigma = \{ \forall x(F(x) \rightarrow G(x)), \exists yF(y), \neg \exists zG(z) \}$

Step 2: Put Σ in the trunk of a new tree.

Step 3: Decompose.

Step 4: Check for closed branches.



Establishment Holds!

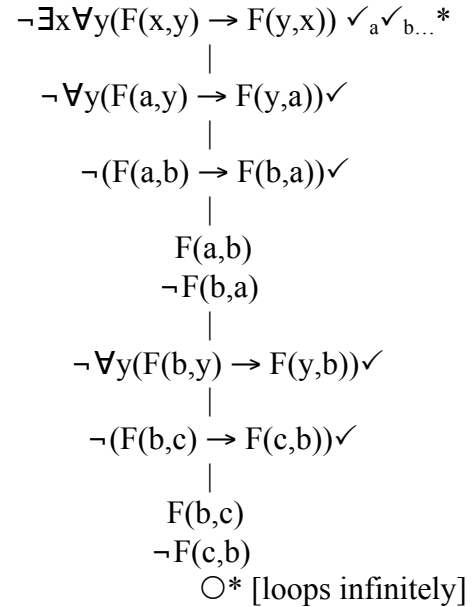
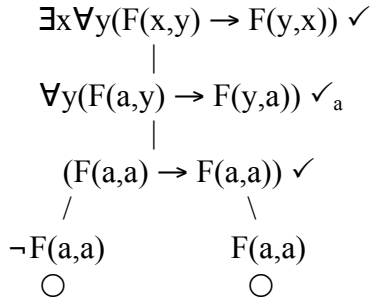
Everything is great, unless you get a **loop**. If you are applying either the universal rule or the negated existential rule in such a way that you are continually reinstantiating on new constants (i.e., FOL names) because the universal or negated existential has an existential or a negated universal below it, the branch is completed iff no new constants can close the branch. In essence, the loop makes the branch infinitely long. But, because of the way the new constants are generated, no incompatibilities will be created. When you notice that this is happening (as in the tree for Σ_2 in the next example), mark the universal (or negated existential) with an asterisk (to indicate that the wff will be continually, provisionally checked to infinity). Then, so long as all of the other wffs on the branch are checked, you can mark the branch as open.

II. Determine whether a Theorem, Anti-theorem, or Neutral: $\exists x \forall y (F(x,y) \rightarrow F(y,x))$

Step 1: Put $\Sigma_1 = \{\phi\}$ in the trunk of a tree, put $\Sigma_2 = \{\neg\phi\}$ in the trunk of a different tree.

Step 2: Decompose in each tree.

Step 3: Check for closed branches.



Answer: $\exists x \forall y (F(x,y) \rightarrow F(y,x))$ is a Neutral!

Practice Problems

Part I. Theorem, Anti-theorem, or Neutral

1. $\forall x (F(x) \rightarrow G(x))$
2. $\exists y (F(y) \wedge F(y))$
3. $(F(a,b) \rightarrow \exists x \exists y F(x,y))$
4. $\forall x \exists y R(x,y)$
5. $\neg \exists x \forall y (R(x,y) \wedge \neg R(x,y))$

Part II. Establishment or Non-establishment

1. $\{ \forall x (F(x) \rightarrow G(x)), \forall y (G(y) \rightarrow H(y,y)), \neg \exists z H(z,z) \} \vdash \neg F(a)$
2. $\{ \forall x R(x,x), \forall x \forall y \forall z ((R(x,y) \wedge R(y,z)) \rightarrow R(x,z)) \} \vdash \forall x \forall y (R(x,y) \rightarrow R(y,x))$

Part III. Compatible or Incompatible

1. $\{ F(a,b), \neg F(b,c), (a = b) \}$
2. $\{ (R(a,b) \wedge \neg R(c,d)), ((a \neq c) \rightarrow (b = d)), (a = e), (c = e) \}$

Part IV. Coupled or Uncoupled

1. $\forall x \forall y ((F(x) \wedge F(y)) \rightarrow (x = y)), \forall z \forall w ((F(z) \rightarrow F(w)) \rightarrow (z = w))$
2. $(F(a) \wedge (a = c)), (F(c) \wedge (a = c))$